Wideband Adaptive Beamforming Using Linear Phase Filterbanks

Peter G. Vouras, Johns Hopkins University
Trac D. Tran, Johns Hopkins University

Key Words: linear phase filter banks, adaptive beamforming, wideband

SUMMARY & CONCLUSIONS

This paper describes the use of linear phase filter banks to perform wideband adaptive beamforming. The application of two types of filter banks is investigated. First, a simple Discrete Fourier Transform Filter Bank (DFTFB) is examined, and second a Linear Phase Paraunitary Filter Bank (LPPUFB) designed using nonlinear optimization techniques is reviewed. For both filter banks, it is shown that computing and applying the optimal beamformer weights independently for different frequency bands yields better performance than a narrowband beamformer.

1. *INTRODUCTION*

Consider a linear array of N sensors and let $\mathbf{x}[k]$ denote the Nx1 vector of signals received by each element in the array. In the context of a radar, the vector $\mathbf{x}[k]$ is given by

$$\boldsymbol{x}[k] = \boldsymbol{s}[k] + \boldsymbol{i}[k] + \boldsymbol{n}[k] \tag{1}$$

where $\mathbf{s}[k]$, $\mathbf{i}[k]$, and $\mathbf{n}[k]$ are statistically independent samples of the target signal, external interference and sensor thermal noise respectively. In spatial beamforming, a complex weight is applied to the output of each array element and then all the weighted signals are summed to form the beamformer output $y[k]$ as in

$$y[k] = \boldsymbol{w}^{\boldsymbol{H}} \boldsymbol{x}[k] . \tag{2}$$

The Linearly Constrained Minimum Variance (LCMV) beamformer minimizes the array output power, J, while constrained to pass a target signal from some direction with a specified gain, and simultaneously place nulls in the direction of interference sources [1]. The objective function to be minimized is,

$$\begin{aligned} &\min_{\boldsymbol{w}} \quad J = \boldsymbol{w}^{\boldsymbol{H}} \boldsymbol{R}_{xx} \boldsymbol{w} \\ &\textit{subject to} \quad \boldsymbol{C}^{\boldsymbol{H}} \boldsymbol{w} = \boldsymbol{f} \end{aligned} \tag{3}$$

where $\mathbf{w}$ is the vector of weights applied to the elements in the array, $\mathbf{R}_{\mathbf{xx}} = E[\mathbf{x}[k]\mathbf{x}[k]^{\mathbf{H}}]$ is the N-by-N data covariance matrix, $\mathbf{C}$ is the N-by-L constraint matrix, and $\mathbf{f}$ is the gain vector. Typically $\mathbf{C}$ and $\mathbf{f}$ have the form

$$\begin{aligned} \boldsymbol{C} &= \begin{bmatrix} \boldsymbol{u}(\varphi_0) & \boldsymbol{u}(\varphi_1) & \cdots & \boldsymbol{u}(\varphi_{L-1}) \end{bmatrix} \\ \boldsymbol{f} &= \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T \end{aligned} \tag{4}$$

where $\mathbf{u}(\varphi_0)$ is the N-by-1 target steering vector defined as

$$\boldsymbol{u}(\varphi_0) = \begin{bmatrix} 1, & e^{-j\varphi_0}, & \ldots, & e^{-j(N-1)\varphi_0} \end{bmatrix}^T, \tag{5}$$

and $\mathbf{u}(\varphi_i)$, $1 \le i \le L$-1, are the interference steering vectors. For a signal impinging on the array from a direction θ_i relative to broadside, the electrical angle φ_i is given by

$$\varphi_i = \frac{2\pi d \sin\theta_i}{\lambda} \tag{6}$$

where d is the spacing between the array elements, and λ is the wavelength. Since $\mathbf{R}_{\mathbf{xx}}$ is not known exactly, it may be estimated from the data as the average of vector outer products taken over some interval,

$$\hat{\boldsymbol{R}}_{xx} = \frac{1}{P} \sum_{i=1}^{P} \boldsymbol{x}[i] \boldsymbol{x}[i]^H . \tag{7}$$

In the Generalized Sidelobe Canceler (GSC) structure shown in Fig. 1, $\mathbf{w}$ is decomposed into a quiescent component, $\mathbf{w_q}$ that resides in the constraint subspace, and an adaptive component, $\mathbf{w_a}$ that is orthogonal to it. Define an N-by-(N-L) matrix $\mathbf{B}$ whose columns form a basis for the orthogonal complement of the space spanned by the columns of the constraint matrix, i.e. $\mathbf{B^H C} = \mathbf{0}$. Then, the optimal solution to (3) is given by

$$\mathbf{w} = \mathbf{w_q} - \mathbf{B}\mathbf{w_a} . \tag{8}$$

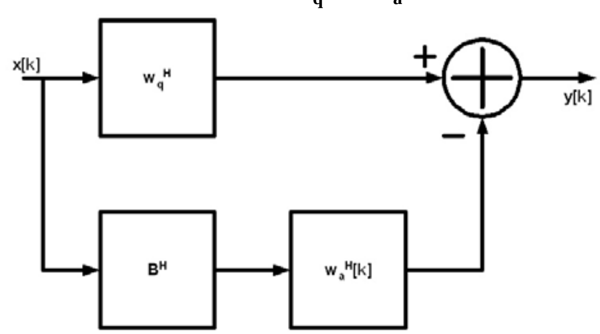

Fig.1. GSC Beamformer

The upper path in the diagram does not vary with time and the quiescent vector $\mathbf{w_q}$ is that part of the weight vector $\mathbf{w}$ that satisfies the linear constraints exactly. The vector $\mathbf{w_q}$ is given by

$$\boldsymbol{w_q} = \boldsymbol{C}\left(\boldsymbol{C}^{\boldsymbol{H}}\boldsymbol{C}\right)^{-1} \boldsymbol{f} . \tag{9}$$

The vector $\mathbf{w_a}$ is the (N-L)-by-1 adaptive weight vector that can vary freely to improve interference suppression in the (N-L)-dimensional subspace orthogonal to the constraint subspace. Define the terms

$$\boldsymbol{p_z} = \boldsymbol{B}^{\boldsymbol{H}} \boldsymbol{R}_{xx} \boldsymbol{w_q}, \quad \boldsymbol{R}_{zz} = \boldsymbol{B}^{\boldsymbol{H}} \boldsymbol{R}_{xx} \boldsymbol{B} , \tag{10}$$

and the adaptive weight vector can be expressed as

$$\boldsymbol{w_a} = \boldsymbol{R}_{zz}^{-1} \boldsymbol{p_z} , \tag{11}$$

which corresponds to the Wiener filter for minimizing the mean square error between the upper and lower branches [2].

A useful figure of merit for adaptive spatial beamformers is the Signal-to-Interference-Plus-Noise Ratio (SINR) defined as

$$SINR = \frac{\boldsymbol{w}^H \boldsymbol{R}_{ss} \boldsymbol{w}}{\boldsymbol{w}^H \boldsymbol{R}_{i+n} \boldsymbol{w}} \tag{12}$$

where $\mathbf{R_{ss}} = E[\mathbf{s}[k]\mathbf{s}[k]^\mathbf{H}]$ and $\mathbf{R_{i+n}} = E[(\mathbf{i}[k]+\mathbf{n}[k])(\mathbf{i}[k]+\mathbf{n}[k])^\mathbf{H}]$ are the N-by-N signal and interference-plus-noise covariance matrices respectively. In the case of a point signal source,

$$\boldsymbol{s}[k] = s[k]\boldsymbol{u}(\varphi_0), \quad R_{ss} = \sigma_s^2 \boldsymbol{u}(\varphi_0)\boldsymbol{u}(\varphi_0)^H \tag{13}$$

where $s[k]$ is the zero-mean signal waveform and $\sigma_s^2 = E[|s[k]|^2]$ is the variance of $s[k]$.

Notice that for a single point target at a known location, the problem

$$\begin{aligned} &\max_{w} \quad SINR \\ &\textit{subject to} \quad \boldsymbol{C}^H \boldsymbol{w} = \boldsymbol{f} \end{aligned} \tag{14}$$

is equivalent to the problem

$$\begin{aligned} &\min_{w} \quad \boldsymbol{w}^H \boldsymbol{R}_{i+n} \boldsymbol{w} \\ &\textit{subject to} \quad \boldsymbol{C}^H \boldsymbol{w} = \boldsymbol{f} \end{aligned} \tag{15}$$

because for every feasible $\mathbf{w}$, the numerator of the SINR remains a constant. Therefore, the GSC formulation applied to solve (15) will also yield the optimal solution for (14).

2. *BAND PARTITIONED BEAMFORMING*

For wideband interference impinging on an array, the computed beamformer weights should be a function of frequency for better performance. By partitioning the received spectrum into contiguous subbands and computing independent weights to be applied to each subband, it is possible to achieve greater SINRs for wideband signals and interference than by simply applying the same spatial weights to all the data, as in a narrowband beamformer. To accomplish the band partitioning, a filter bank such as the one illustrated in Fig. 2 may be utilized.

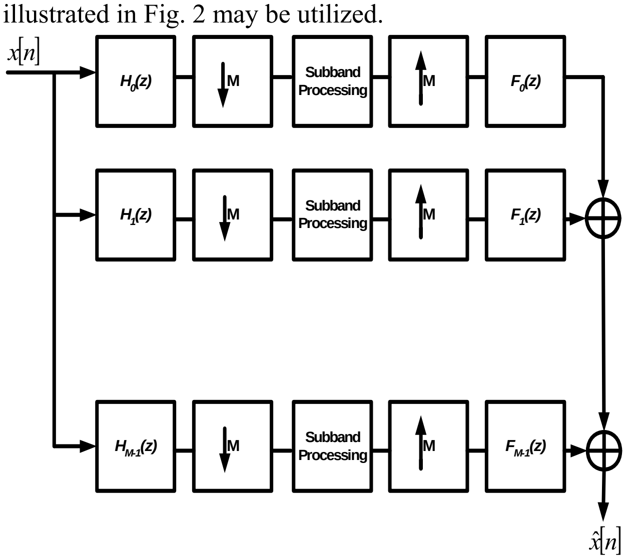

Fig. 2. Maximally Decimated Filter Bank

The general theory of filter banks is described in [3] and will be briefly summarized here. The filters $H_k(z)$ form the analysis section of the filter bank and the filters $F_k(z)$ form the synthesis section. The output of each analysis filter is decimated by a factor M, after which independent processing can be performed in each of the subbands. The subband processing in this case consists of computing and applying the beamformer weights as in (8) and (2). After the subband processing is complete, the subband data is expanded by a factor M and convolved with the synthesis channel filter. Then all the channel outputs are summed to yield the final output $\hat{x}[n]$. If, in the absence of subband processing, the input $x[n]$ equals $\hat{x}[n]$, then the filter bank is said to achieve perfect reconstruction (PR). If the decimation factor M is equal to the number of channels, then the filter bank is referred to as maximally decimated because the subband signals are sampled at the lowest possible sampling rate.

A much more computationally efficient implementation of the same filter bank using polyphase matrices is illustrated in Fig. 3. Each analysis filter $H_k(z)$ with impulse response $h_k(n)$ can be expressed in terms of its polyphase components as

$$H_k(z) = \sum_{l=0}^{M-1} z^{-l} E_{kl}(z^M), \quad k = 0, \ldots, M-1 \tag{16}$$

where

$$e_{kl}(n) = h_k(l + Mn), \quad 0 \le l \le M-1 \tag{17}$$

and

$$E_{kl}(z) = \sum_{n=-\infty}^{\infty} e_{kl}(n) z^{-n}. \tag{18}$$

Similarly, each synthesis filter can be expressed in terms of its polyphase components $R_{lk}(z)$ as

$$F_k(z) = \sum_{l=0}^{M-1} z^{-(M-1-l)} R_{lk}(z^M), \quad k = 0, \ldots, M-1. \tag{19}$$

Now define the M-by-M polyphase component matrices $\boldsymbol{E}(z)$ and $\boldsymbol{R}(z)$ as

$$\boldsymbol{E}(z) = [E_{kl}(z)], \quad \boldsymbol{R}(z) = [R_{lk}(z)]. \tag{20}$$

The polyphase matrix evaluated for $z = e^{j\omega}$ is denoted by $\boldsymbol{E}(\omega)$ or $\boldsymbol{R}(\omega)$. The frequency responses of the analysis bank filters may be written as

$$\boldsymbol{h}(\omega) = \boldsymbol{E}(\omega M)\boldsymbol{s}(\omega), \tag{21}$$

where the analysis-bank transfer function vector $\mathbf{h}(\omega)$ and the delay chain vector $\mathbf{s}(\omega)$ are given by

$$\boldsymbol{h}(\omega) = \begin{bmatrix} H_0(\omega) \\ H_1(\omega) \\ \vdots \\ H_{M-1}(\omega) \end{bmatrix}, \quad \boldsymbol{s}(\omega) = \begin{bmatrix} 1 \\ e^{-j\omega} \\ \vdots \\ e^{-j(M-1)\omega} \end{bmatrix}. \tag{22}$$

Similarly, the frequency responses of the synthesis bank filters may be written as

$$\boldsymbol{f}^T(\omega) = e^{-j(M-1)\omega} \boldsymbol{s}^H(\omega) \boldsymbol{R}(\omega M), \tag{23}$$

where the synthesis-bank transfer function vector $\mathbf{f}(\omega)$ is given by

$$\boldsymbol{f}^T(\omega) = \begin{bmatrix} F_0(\omega) & F_1(\omega) & \cdots & F_{M-1}(\omega) \end{bmatrix}. \tag{24}$$

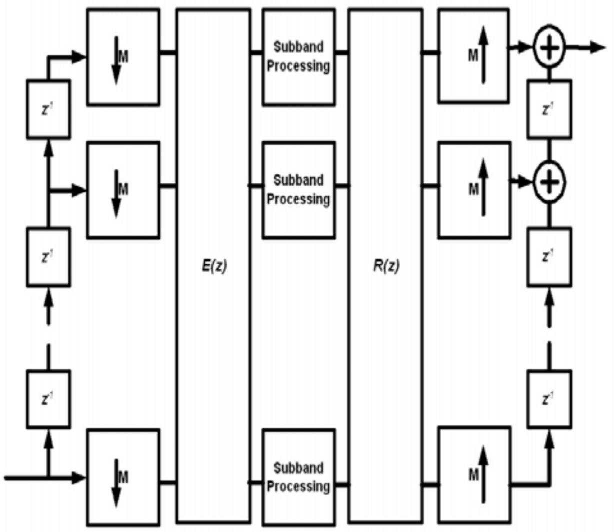

Fig. 3. Polyphase Implementation of Filter Bank

The most obvious way to implement band partitioned adaptive beamforming is to place an analysis and synthesis filter bank at each array element. Then the adaptive beamformer weights could be computed and applied within each subband processor. After the frequency subbands are recombined by the synthesis filter bank at each element, all the element data could be spatially combined in a conventional beamformer that coherently sums all the array element signals. This architecture is illustrated in Fig. 4. A more efficient yet equivalent architecture is illustrated in Fig. 5. In this architecture all the frequency subbands are spatially combined after the adaptive beamformer weights are applied. Then only one synthesis filter bank is necessary to recombine the frequency subbands and generate the final output. All the data presented in this paper will be for the architecture shown in Fig. 5.

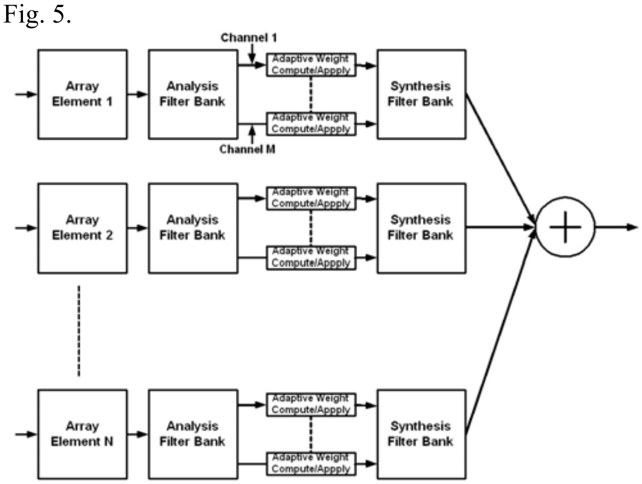

Fig. 4. Band Partitioned Beamforming Architecture 1

To preserve the incremental phase progression of the received signal across the array elements and to avoid introducing any phase distortions into the filter bank channels that might affect the beamformer weight computations, it is necessary for all the analysis and synthesis filters in the architecture to have linear phase. Two PR filter banks with linear phase filters are the Discrete Fourier Transform Filter Bank (DFTFB) and the Linear Phase Paraunitary Filter Bank (LPPUFB), each of which will be briefly described next.

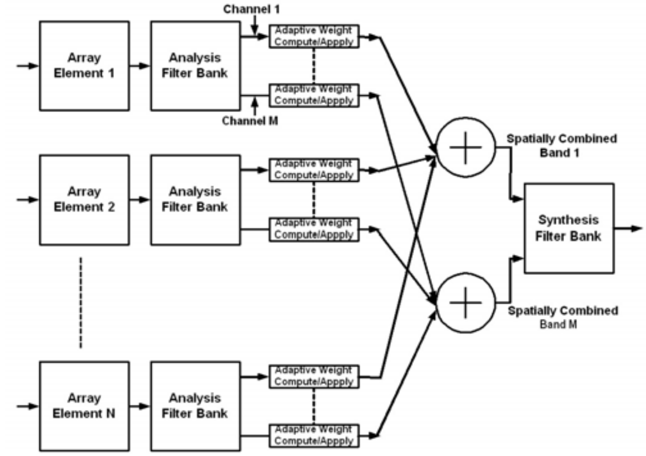

Fig. 5. Computationally Efficient Beamforming Architecture 2

3. *DISCRETE FOURIER TRANSFORM FILTER BANK*

A DFTFB as shown in Fig. 6 has the polyphase matrices

$$\boldsymbol{E}(\omega) = \boldsymbol{F}_{\boldsymbol{M}}^{*}, \quad \boldsymbol{R}(\omega) = \boldsymbol{F}_{\boldsymbol{M}} . \tag{25}$$

Here the matrix $\mathbf{F_M}$ is the DFT matrix and $\mathbf{F_M}^*$ is its conjugate. All M filters in the filter bank are obtained from a single prototype filter, $G_0(\omega)$, through the relationship,

$$G_k(\omega) = G_0\left(\omega - 2\pi k/M\right), \quad 0 \le k \le M-1 . \tag{26}$$

Thus, the frequency responses of the M filters are obtained from that of $G_0(\omega)$ by right shifts of $2\pi k/M$ in the frequency domain. The frequency response of $G_0(\omega)$ is,

$$G_0(\omega) = 1 + e^{-j\omega} + \cdots + e^{-j\omega(M-1)} = D(\omega, M)\, e^{-j0.5\omega(M-1)} \tag{27}$$

where $D(\omega, M)$ is the Dirichlet kernel. Since $G_0(\omega)$ has linear phase, all the filters in the filter bank have linear phase. Fig. 6 illustrates the frequency responses of the channel filters for the case where there are 32 channels in the filter bank and all the filters have length 32.

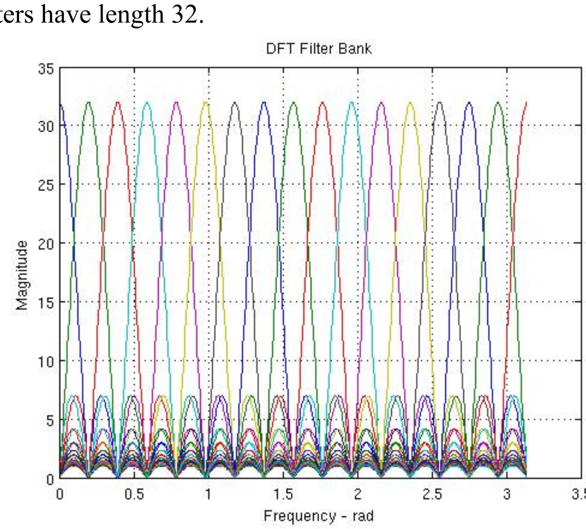

Fig. 6. DFT Filter Bank Response

4. *LINEAR PHASE PARAUNITARY FILTER BANK*

A Paraunitary (PU) filter bank is a PR filter bank which satisfies the additional property that

$$\boldsymbol{E}(\omega)\boldsymbol{R}(\omega)=\boldsymbol{I}, \quad \boldsymbol{E}(\omega)=\boldsymbol{R}^{\boldsymbol{H}}(\omega) \quad \forall\omega . \tag{28}$$

In a LPPUFB all the filters are either symmetric or anti-symmetric, i.e.

$$h_k(L-1-n)=d_k h_k(n) \tag{29}$$

where L is the length of the filter and d_k is 1 for symmetric filters and -1 for anti-symmetric filters. Also, the synthesis filters and the analysis filters are time-reversed versions of each other so,

$$f_k(n)=h_k(L-1-n) . \tag{30}$$

According to [4], the symmetry condition in equation (29) holds if and only if the polyphase matrix satisfies

$$\boldsymbol{E}(z)=z^{-(K-1)}\boldsymbol{D}\boldsymbol{E}\left(z^{-1}\right)\boldsymbol{J} \tag{31}$$

where $\mathbf{D} = \mathrm{diag}(d_0, d_1, \ldots, d_{M-1})$, K–1 is the order of the polyphase matrix, the length of the channel filters is MK and

$$\boldsymbol{J}=\begin{bmatrix} 0 & 0 & 1 \\ 0 & \iddots & 0 \\ 1 & 0 & 0 \end{bmatrix} . \tag{32}$$

The order of the polyphase matrix is used to denote the highest power of z^{-1} in the matrix. The linear phase PU filterbanks considered here have real coefficients and M is assumed to be even. For even M, there must be $M/2$ symmetric filters and $M/2$ anti-symmetric filters.

As described in [4], if $\mathbf{E}(z)$ is an M-by-M real causal finite impulse response (FIR) transfer function matrix with order not exceeding K-1, then it is PU and satisfies the linear phase condition (31) if and only if there exist $M/2$-by-$M/2$ orthogonal matrices $\mathbf{V_0}$ and $\mathbf{W_k}$ so that $\mathbf{E}(z)$ can be factored as

$$\boldsymbol{E}(z)=\frac{\sqrt{2}}{2^K}\prod_{k=K-1}^{1}\left\{\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{W_k}\end{bmatrix}\boldsymbol{Q}\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{0} & z^{-1}\boldsymbol{I}\end{bmatrix}\boldsymbol{Q}\right\}\begin{bmatrix}\boldsymbol{V}_0 & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{W}_0\end{bmatrix}\boldsymbol{P}$$

where

$$\boldsymbol{Q}=\begin{bmatrix}\boldsymbol{I} & \boldsymbol{I}\\ \boldsymbol{I} & \boldsymbol{-I}\end{bmatrix}, \quad \boldsymbol{P}=\begin{bmatrix}\boldsymbol{I} & \boldsymbol{J}\\ \boldsymbol{I} & \boldsymbol{-J}\end{bmatrix} . \tag{34}$$

Using this factorization theorem it is possible to design an LPPU filter bank using nonlinear optimization techniques to match specified criteria by parameterizing the orthogonal matrices $\mathbf{V_0}$ and $\mathbf{W_k}$ in terms of rotation angles. Specifically, each matrix $\mathbf{V_0}$ and $\mathbf{W_k}$ is the product of ½M(M-1) Givens rotation matrices of the form,

$$\boldsymbol{S_{ij}}(\theta_k)=\begin{bmatrix} 1 & 0 & 0 & 0 & 0\\ 0 & \cos(\theta_k) & \cdots & \sin(\theta_k) & 0\\ 0 & \vdots & 1 & \vdots & 0\\ 0 & -\sin(\theta_k) & \cdots & \cos(\theta_k) & 0\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}, \tag{35}$$

$$1\le k\le \tfrac{1}{2}M(M-1)$$

where $\cos(\theta_k)$ is placed in the ith row and ith column, $\sin(\theta_k)$ is in position (i, j), $-\sin(\theta_k)$ is in position (j, i), and $\cos(\theta_k)$ is in position (j, j). The order of the matrices $\boldsymbol{S}_{ij}$ in the product is important. In general, an M-by-M orthogonal matrix $\mathbf{G_M}$ can be decomposed into the following product sequence of Givens rotation matrices,

$$\boldsymbol{G}_M=\left\{\boldsymbol{S}_{M-2,M-1}\right\}\cdots\left\{\boldsymbol{S}_{1,M-1}\cdots\boldsymbol{S}_{12}\right\}\left\{\boldsymbol{S}_{0,M-1}\cdots\boldsymbol{S}_{01}\right\} . \tag{36}$$

One optimization criterion for designing the LPPUFB is to minimize the energy in the stopbands of the filters. By doing so, the aliasing components that leak into adjacent subbands will be minimized and thereby improve the accuracy of the adaptive weight calculations. Thus, the objective function to be minimized for designing a LPPUFB becomes

$$\eta=\sum_{i=0}^{M-1}\int_{\omega\in stopband}\left|H_i(\omega)\right|^2 d\omega . \tag{37}$$

Using the transfer function matrix $\mathbf{E}(\omega)$, each individual analysis filter may be described by

$$H_i(\omega)=\boldsymbol{e}_i^T\boldsymbol{E}(\omega)\boldsymbol{s}(\omega) \tag{38}$$

where $\mathbf{e_i}$ is a vector of all zeros except for a one in the ith component and $\mathbf{s}(\omega)$ is the delay chain vector. Since the synthesis filters are time reversed versions of the analysis filters, they have the same magnitude response.

By embedding the factorization (33) of the transfer function matrix into the objective function (37), the solution filter bank is guaranteed to be PU with linear phase filters. Upon substituting the decomposition for $\mathbf{E}(\omega)$ into η, there will be K+1 rotation angles θ_k that are free parameters. To restrict the value of each θ_k to lie between -2π and 2π, define the following penalty function,

$$\varphi=\sum_{j=1}^{K+1}\left(\max\left\{0,\theta_j-2\pi\right\}\right)^2+\left(\max\left\{0,-2\pi-\theta_j\right\}\right)^2 . \tag{39}$$

Now one can solve the unconstrained minimization problem,

$$\min \quad \xi=\eta+\alpha\varphi \tag{40}$$

where α is a positive scalar. One of the more popular techniques for solving unconstained minimization problems is the Broyden, Fletcher, Goldfarb, and Shanno (BFGS) algorithm, which falls under the category of a quasi-Newton method. The MATLAB function *fminunc* implements a version of the BFGS algorithm for unconstrained minimization. Fig. 7 illustrates the designed LPPUFB for the case where M=4.

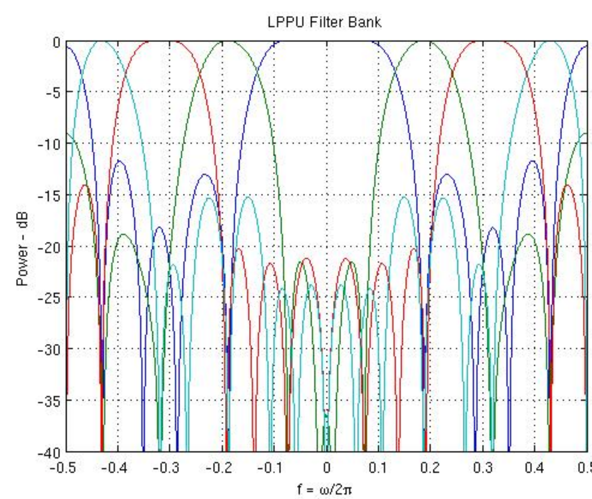

Fig. 7. LPPUFB

5. *RESULTS AND CONCLUSION*

The performance of the band partitioned adaptive beamformer using a 32-channel DFTFB with 32 tap filters is shown in Fig. 8 for a linear array populated with 10 elements. The Signal-to-Noise Ratio (SNR) at each element is 10 dB and the Jamming-to-Noise Ratio (JNR) at each element is 30 dB. The target was assumed to be located at array broadside and the interference source at 44 degrees azimuth. No signal samples were assumed in the training data. Plotted is the SINR as a function of the fractional bandwidth, f_b [5]. Both signal and interference have the same fractional bandwidth, which is defined as the ratio of the impinging signal bandwidth to the array center frequency. Wide bandwidth signals correspond to f_b = 0.5. For comparison, the figure includes the SINR plot for a conventional narrowband beamformer and also illustrates how SINR improves with increasing number of channels in the filter bank.

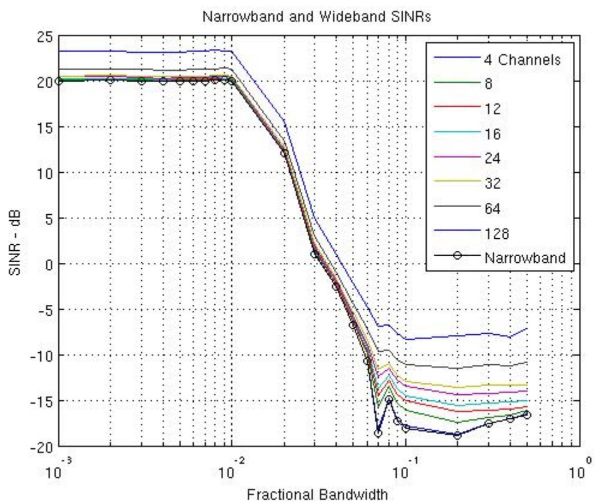

Fig. 8. Beamformer Performance – DFTFB

Fig. 9 illustrates the SINR performance of a band partitioned beamformer using a LPPUFB for the case where *M*=4 and compares it to the DFTFB beamformer. As is clear from the figure, the LPPUFB beamformer performs comparably to the DFTFB beamformer.

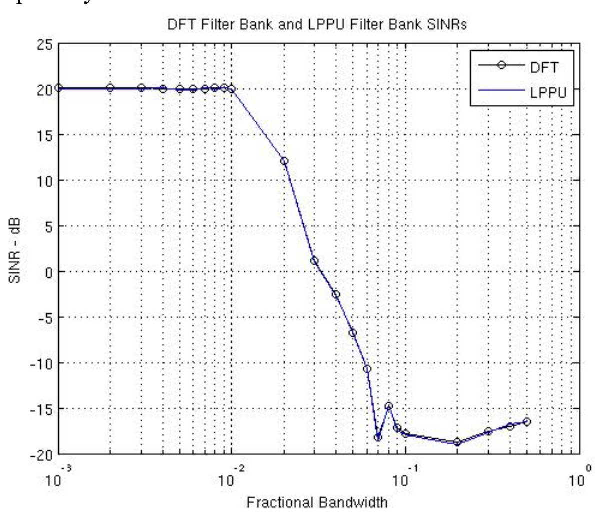

Fig. 9. Beamformer Performance - LPPUFB

Fig. 10 illustrates the array beam pattern for one of the subbands of the 32 channel DFTFB beamformer and overlays the beam pattern produced by a uniformly weighted array. Essentially, the overall array response using a band partitioned beamformer will be a function of frequency as well as azimuth, as illustrated by Fig. 11. The vertical scale in Fig. 11 is in units of dB normalized to the peak response.

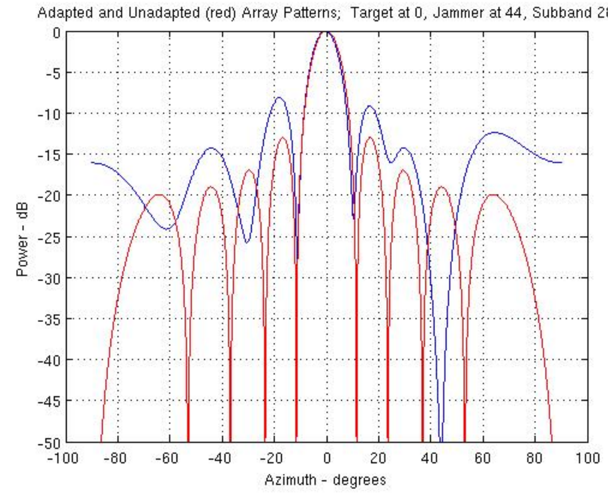

Fig. 10. Array Pattern – Subband 28

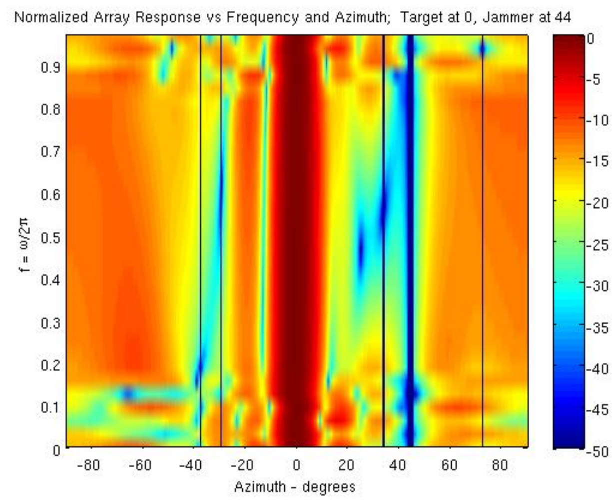

Fig. 11. Overall Array Response

6. *REFERENCES*

1. S. Haykin, *Adaptive Filter Theory*, Prentice Hall, Upper Saddle River, NJ, 1996.
2. Z. Tian, "A Recursive Least Squares Implementation for LCMP Beamforming Under Quadratic Constraint", *IEEE Trans. on Signal Processing*, Vol. 49, No. 6, June 2001.
3. P. P. Vaidyanathan, *Multirate Systems and Filter Banks*, Prentice Hall, NJ, 1993.
4. X. Gao, T. Nguyen, and G. Strang, "On Factorization of M-Channel Paraunitary Filterbanks", *IEEE Trans. on Signal Processing*, Vol. 49, No. 7, July 2001.
5. P. Cahill, K. Gerlach, and F. Lin, "A Method of Sidelobe Cancellation Using Wavelet Packets", Proc. IEEE Radar Conf., Philadelphia, PA, April 26-29 2004.