ADAPTIVE TRANSMIT NULLING USING WAVEFORM EMBEDDED DITHERS

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ABSTRACT

This paper describes a novel concept for performing adaptive transmit nulling in radars. Typically, the average depth of open-loop transmit nulls is limited by the Error Sidelobe Level (ESL) of an array. The ESL is determined by random errors in the array manifold, such as the residual calibration errors. Theoretically, it is possible to null below the ESL by using the backscatter from a target to create a null adaptively. The objective function to be minimized in this case is the power of the received backscatter in the direction of the target, which is assumed to be the same as the desired null direction. A potential drawback to this nulling approach would be long radar dwell times on the target to estimate the gradient of the objective function.

To accelerate the convergence of a closed-loop transmit nulling algorithm, this paper proposes a theoretical technique for estimating the gradient of the array beamformer output in near real time by embedding narrowband sinusoidal dithers in the transmitted waveform. Dithers are externally applied disturbances that have been used in control system and signal processing applications to mitigate the effects of nonlinearity, hysteresis, and quantization [1, 2]. Typically the dithers applied are wideband random signals, essentially additive noise. The dithers described in this paper are different however in that they are narrowband sinusoids. The proposed dithering technique has been successfully applied to optical communication links to achieve adaptation speeds on the order of microseconds, [3] – [7]. The simulated results in this paper are intended to demonstrate the feasibility of using narrowband dithers to perform adaptive transmit nulling in near real time on radars.

Index Terms— narrowband dithers, transmit nulling, adaptive beamforming, gradient estimation

1. CONCEPT OF OPERATIONS

The goal of the radar in the assumed operational scenario is to scan a search volume using the main beam while maintaining a transmit null towards a fixed or slowly varying direction in space corresponding to a target. The desired null direction is taken to be within the sidelobe region of the array transmit pattern and the desired null depth is below the ESL of the radar. The transmit null will be formed in space by the coherent combination of the main beam antenna pattern and an auxiliary antenna pattern. The auxiliary antenna is located on the periphery of the main array and is adaptively weighted by a complex magnitude.

To determine the auxiliary antenna weight, the radar transmits the main beam in a given search direction, and then scans the receive beam towards the desired null direction to measure the backscatter from the target. This sequence may be performed in real time in the field using an airborne target of interest, or it may be performed on a test range using a controlled target. The target must be close enough to the radar to ensure a sufficiently high signal-to-noise ratio until the algorithm converges to a null depth below the ESL.

By using a steepest descent algorithm, the radar computes over several pulses the complex auxiliary antenna weight necessary to minimize the received array output power in the direction of the desired null. On transmit, the combination of a weighted auxiliary antenna pattern and the main beam transmit pattern forms a directional null in a manner analogous to the null formed on receive by a sidelobe canceller in the direction of interference. The nulling process must be repeated for every new main beam steering direction or whenever the target crosses into a new transmit sidelobe. To implement this nulling technique, the power output and the directivity of the auxiliary antenna must be sufficiently high to cover the radiated power in the sidelobes of the main array transmit pattern.

2. ARRAY CONFIGURATION

For the results presented in this paper, a large phased array with an auxiliary antenna was simulated. The location of the auxiliary antenna relative to the main array is shown in Fig. 1. The position of each element is measured relative to the center of the main array. The main array is roughly elliptical and has approximately 5400 elements that are placed on a triangular grid with half-wavelength spacing between adjacent elements. The operating frequency of the array is assumed to be at X-band. The auxiliary antenna is a rectangular array with 61 elements, also laid out on a triangular grid. An azimuth principal plane cut of the auxiliary antenna pattern is shown in Fig. 2.

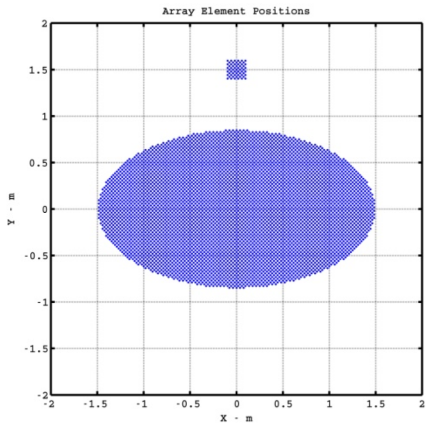

Fig. 1. Relative Location of Auxiliary Antenna

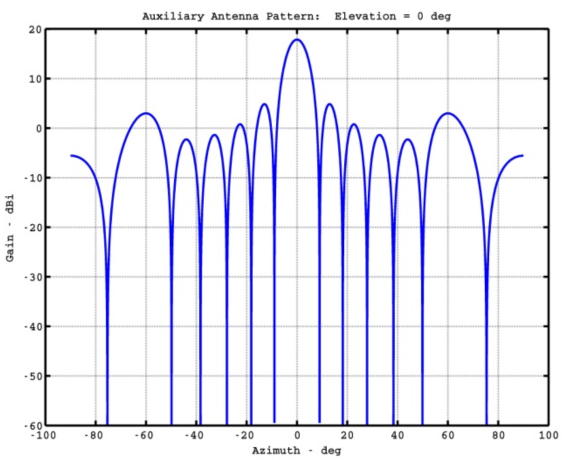

Fig. 2. Auxiliary Antenna Pattern

3. GRADIENT SEARCH

At time instant t_n, corresponding to transmitted pulse n, the objective of the transmit nulling algorithm is to compute a complex weight w that minimizes the expected power, $J(n;\varphi)$, of the coherent combination of the auxiliary antenna pattern, $A(n;\varphi)$, and the main array pattern, $M(n;\varphi)$, in the direction φ of the desired null. In other words, the algorithm seeks to find a complex scalar w so as to minimize the cost function J,

$$\underset{w}{\text{minimize}}\ J(n;\varphi) = E\left[\left| M(n;\varphi) + wA(n;\varphi)\right|^2\right],$$

where E is expected value. Since the null direction φ remains fixed, it will be dropped from subsequent notation. For simplicity, the transmitted waveform for both the auxiliary and the main antennas is assumed to be the same. It is a real, constant-amplitude, rectangular pulse which is scaled by the complex magnitude of the antenna patterns in the direction of the null.

A steepest descent algorithm may be used to converge to the optimal solution by updating the adaptive weight after each transmitted pulse

$$w(n+1) = w(n) - \mu \nabla_w J(n). \tag{1}$$

The step-size parameter μ is a small positive real constant, appropriately chosen to ensure algorithm convergence. If the value of μ is too large, the iterations will diverge to infinity. If μ is chosen too small, the algorithm may converge over unrealistically many pulses.

4. NARROWBAND DITHERING TECHNIQUE

The primary drawback of an adaptive nulling technique that relies on estimating the gradient of the cost function is its absolute speed of convergence, measured in seconds. However, it is possible to apply independent sinusoidal dithers to the real and imaginary parts of the complex weight w to estimate the gradient in near real time. These dithers would be embedded in the waveform transmitted by the auxiliary antenna until the algorithm converges. Once the algorithm converges, the optimal complex weighting for the auxiliary antenna is known, and the transmitted dithers are no longer necessary.

To derive the proposed approach, consider any objective function $J(\boldsymbol{u})$, where $\boldsymbol{u}$ is an Nx1 vector of control variables. For each component of $\boldsymbol{u}$, superimpose a sinusoidal dither θ of differing frequency, as in

$$\boldsymbol{u}' = \boldsymbol{u} + \boldsymbol{\theta} = \boldsymbol{u} + \alpha\left[\cos(\omega_1 t),\quad \cos(\omega_2 t), \ldots, \cos(\omega_N t)\right]^T$$

where α is a small scalar. Using the Taylor series expansion of $J(\boldsymbol{u})$ yields,

$$J(\boldsymbol{u}') = J(\boldsymbol{u}+\boldsymbol{\theta}) = J(\boldsymbol{u}) + \nabla J(\boldsymbol{u})^T \boldsymbol{\theta} + \tfrac{1}{2}\boldsymbol{\theta}^T \nabla^2 J(\boldsymbol{u})\boldsymbol{\theta} + \ldots$$

$$= J(\boldsymbol{u}) + \alpha \sum_{i=1}^{N} \left.\frac{\partial J}{\partial u_i}\right|_{\boldsymbol{u}} \cos(\omega_i t) + \ldots .$$

The components of the gradient vector can now be determined exactly by multiplying $J(\boldsymbol{u}')$ and $\cos(\omega_j t)$, for j = 1, 2,…, N. The result after using trigonometric identities is

$$J(\boldsymbol{u}')\cos(\omega_j t) = J(\boldsymbol{u})\cos(\omega_j t) + \frac{\alpha}{2}\left.\frac{\partial J}{\partial u_j}\right|_{\boldsymbol{u}}$$

$$+ \frac{\alpha}{2}\left.\frac{\partial J}{\partial u_j}\right|_{\boldsymbol{u}} \cos(2\omega_j t) + \frac{\alpha}{2}\sum_{i\neq j}\left.\frac{\partial J}{\partial u_i}\right|_{\boldsymbol{u}} \cos([\omega_i - \omega_j]t)$$

$$+ \frac{\alpha}{2}\sum_{i\neq j}\left.\frac{\partial J}{\partial u_i}\right|_{\boldsymbol{u}} \cos([\omega_i + \omega_j]t) + H.O.T.$$

Observe that the only constant on the right side of the equation is the jth component of the gradient vector scaled by the factor $\alpha/2$. It can therefore be recovered exactly by low-pass filtering the term $J(\boldsymbol{u}')\cos(\omega_j t)$. Once the entire gradient vector is reconstructed, it can be used in the steepest descent algorithm to compute the next iteration of the weight vector.

5. SIMULATED RESULTS

The steepest-descent adaptive-nulling algorithm was simulated for the radar configuration shown in Fig. 1. The main beam was steered to broadside and the desired null direction was chosen to be 9.5° azimuth and 0° elevation. One sinusoidal dither at 100 Hz was applied to the real part of the auxiliary weight w, and an independent dither at 200 Hz was applied to the imaginary part. Both dithers were applied simultaneously. The dithering constant α was chosen to be 0.1. The low pass filter was implemented as a sliding window average of the data using a mask of 128 samples normalized to have unity DC gain. The value of the algorithm step-size parameter μ was chosen to be 0.01. For simplicity, the signal-to-noise ratio of the target returns was assumed constant over the algorithm iterations.

Figure 3 illustrates the received power $J(n)$ as a function of the transmitted pulse number. This plot shows that as the transmit null is being formed, the received power in the direction of the null steadily decreases. After 1000 pulses, a deep null is evident in the transmit pattern.

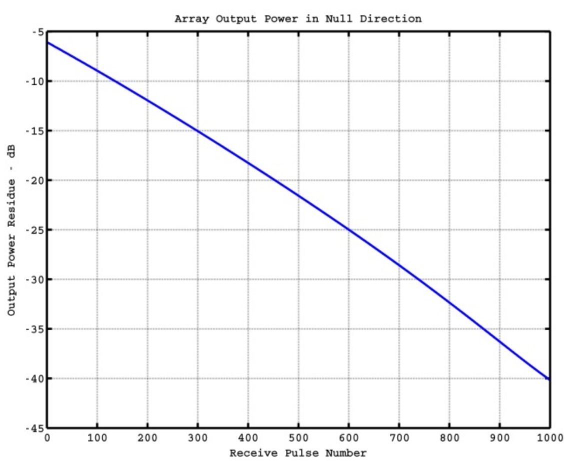

Fig. 3. Received Power in Null Direction

The limit point of the gradient search algorithm will depend on the initial conditions chosen, because local minima in the objective function are possible. Thus, different random values for the initial auxiliary antenna weight w_0 may not all yield the same solution w_M after M iterations. However, a null will still be formed.

As the iterations of the steepest descent algorithm approach a stationary point for J, the gradient approaches zero as shown in Fig. 4. However the stationary points of J are not necessarily stable equilibrium points of the closed-loop nulling system, in which case further iterations will move w away from its optimal value, and the final null depth may be higher than optimal [7].

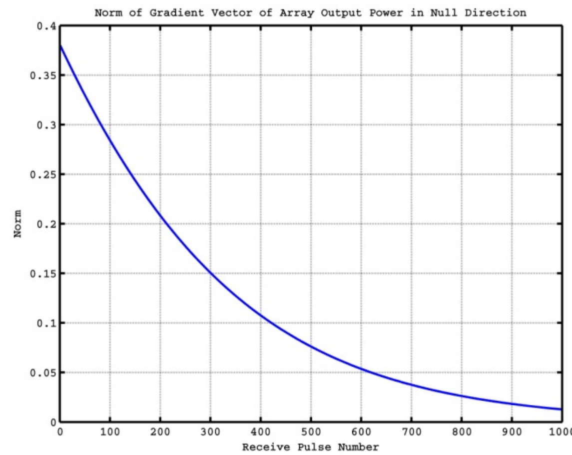

Fig. 4. Evolution of Gradient

Figure 5 shows the transmit null formed after 1000 iterations and compares it to the transmit sidelobe in the unadapted ambient transmit pattern. As the plot illustrates, the null is formed at precisely the desired direction of 9.5° azimuth. However, the overall antenna pattern has been distorted and the sidelobe levels have been raised elsewhere, as shown in Fig. 6. This distortion may or may not be an issue, depending on the radar operational scenario.

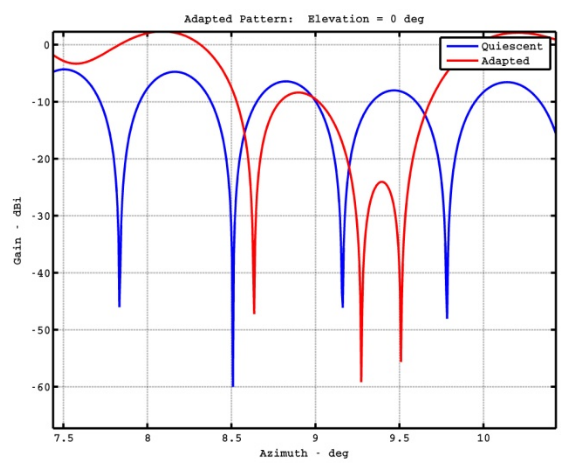

Fig. 5. Close-Up of Adapted Transmit Pattern

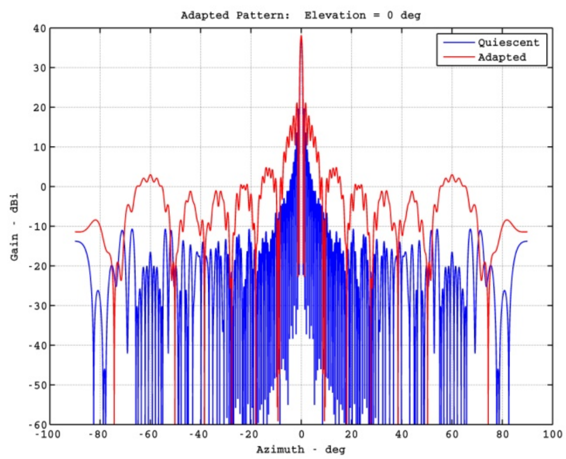

Fig. 6. Comparison of Antenna Patterns

6. NOISE IMMUNITY

A variation to the weight update step (1) is possible which provides some immunity to noise in the feedback loop between the target and radar. The new weight update step becomes [6]

$$w(n+1) = w(n) - \mu \,\mathrm{sgn}\left(\nabla_w J(n)\right).$$

This new step requires only the sign of the gradient components at each iteration. Using this update step, the nulling algorithm will trend towards an optimal value, but not necessarily converge. Instead it may exhibit small oscillations, as shown in Figs. 7 and 8. Nevertheless, computed values for w will still form a desirable transmit null as shown in Figs. 9 and 10.

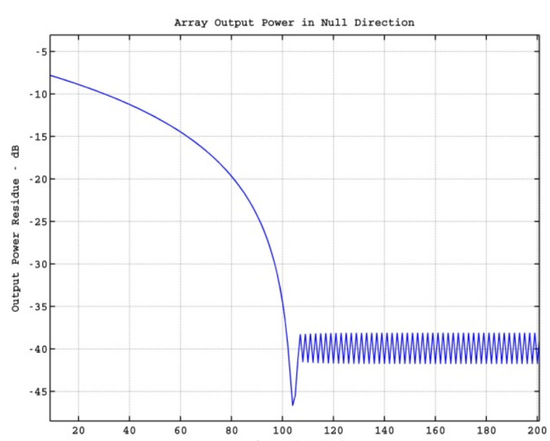

Fig. 7. Received Power in Null Direction; Signum Algorithm

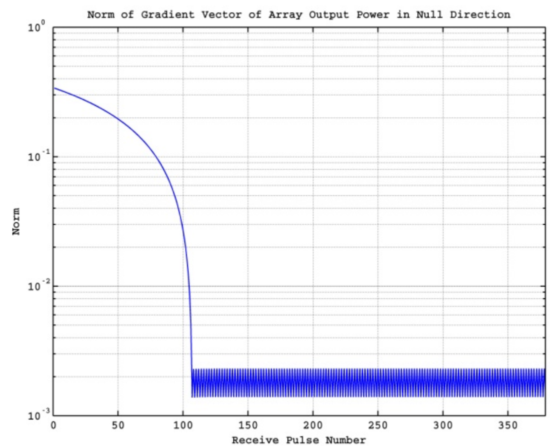

Fig. 8. Evolution of Gradient; Signum Algorithm

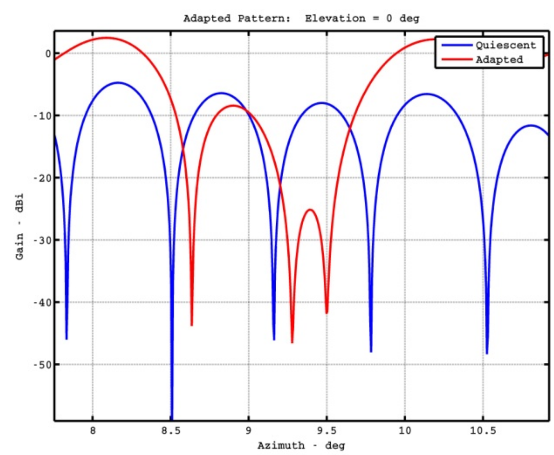

Fig. 9. Close-Up of Adapted Transmit Pattern; Signum Algorithm

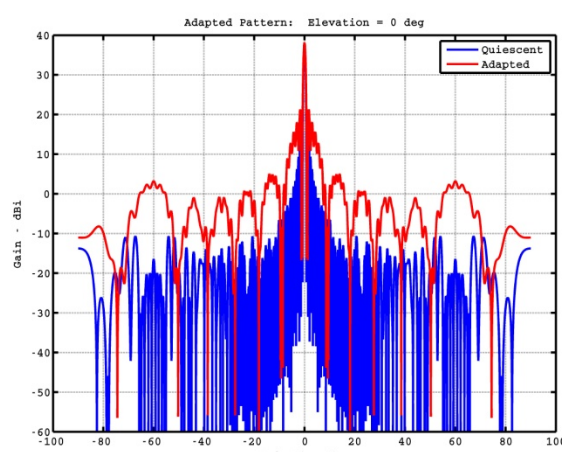

Fig. 10. Comparison of Antenna Patterns; Signum Algorithm

7. PHASE ONLY SOLUTION

If the auxiliary antenna lacks amplitude control but has phase control, it is still possible to form a transmit null by dithering the phase of w. Figure 11 illustrates the transmit null formed when the magnitude of w is held to unity but its phase is allowed to vary. The phase-only null is comparable in depth to the null formed when both the amplitude and phase of w are allowed to vary.

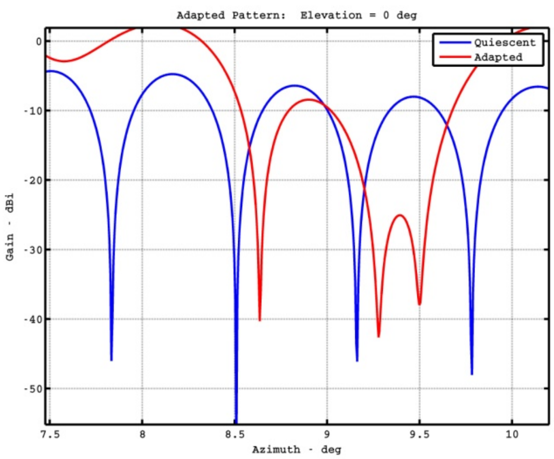

Fig. 11. Comparison of Antenna Patterns; Phase Only Solution

8. CONCLUSIONS

In this paper, a new technique is proposed which adaptively computes a complex weight for an auxiliary antenna to form a spatial null in the transmit pattern of a phased array. The auxiliary antenna weight is computed using a gradient steepest descent algorithm. By embedding independent narrowband sinusoidal dithers in the waveform transmitted by the auxiliary, it is possible to estimate the gradient of the array output power in the desired direction of the null using very few pulses, and then to form the transmit null quickly.

9. REFERENCES

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